Elliptic model for the consideration of the anisotropy by the fatigue criteria: application to wood material

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ABSTRACT

Wood is a material widely used in construction. The determination of its properties is very difficult because of its anisotropy. A presentation of both the physical and mechanical properties of wood has been made in order to understand and further explain the mechanisms responsible for its anisotropy. The modeling of the anisotropy has been done at the level of the stress limits. We assume that they form an ellipse call the anisotropy ellipse, for anisotropic materials and a circle for isotropic materials. The fatigue limit which appears in the expression of the Fogue-Bahuaud criterion was replaced by the equation of the ellipse obtained. A comparison with the literature results and an application on two species of tropical timber, the Azobe and Bilinga, were used to validate the results.

Keywords: Anisotropy, Fatigue criteria, Wood

1 INTRODUCTION

Most components or mechanical structures are now calculated and verified to fatigue by fatigue criteria, once they are subjected in service to various loads. The tendency is to generalize this approach. The consultants who design mechanisms therefore cannot ignore the dimensioning of parts with respect to their resistance to fatigue.

Like many other materials, wood has always been a material of great importance to man. Its use has increased due to the multiplicity of its application. As a natural material, by changes in its characteristics, wood reflects a variety of physical conditions that have surrounded its growth. Its qualities vary depending on the direction of fibers and differ from one species to another. This fluctuation called anisotropy makes it difficult to measure its properties.

The integration of the anisotropy is made in the Fogue Bahaud criterion, by replacing the strain limit in symmetrical alternating traction appearing in the indicator of damage with a function of elliptical anisotropy. The integration of the anisotropy is made in the test Fogue-Bahaud, replacing the fatigue limit in symmetrical alternating traction appears in the damage indicator by a function of elliptical anisotropy whose curve is compared to that Soh Fotsing.

2 START-OFF POINT

To account for the anisotropic fatigue behavior, the following can be envisaged: the incorporation of the anisotropic nature in the law of localization [1] or the taking in account of the orientation of the microstructure in the expression of the fatigue criterion [2].

2.1 Fatigue criterion Fogue-Bahuaud

A literature review [3-6], shows that there are many criteria of multi-axial fatigue grouped into three categories: the first category is made of criteria called empirical, the second which is the critical plan approach criteria and the third which is the global plan approach criteria.

This same literature shows that the criteria for global plan approach gives better results than the critical plan criteria. Among the criteria for comprehensive plan approach, it is shown in Kenmeugne [3] and Soh Fotsing [7] that the criterion Fogue-Bahuaud gives results closer to reality and is suitable for wood material. Moreover, it is very efficient when the main directions of stresses are mobile, which is often the case in the amplitude multi-axial fatigue variable.

The fatigue function criterion is the square root of all the indicators of damage on all the possible facets around the point where the test is applied. It is defined by:

$$\boldsymbol{E_{FB}} = \sqrt{\frac{1}{S}\int_S \ \boldsymbol{E_h^2 dS}} \qquad (1)$$

With $$\boldsymbol{E_h} = \frac{a\tau_{ha}+b\sigma_{hha}+d\sigma_{hhm}}{\sigma_{-1}(N)} \qquad (2)$$

$\boldsymbol{E_h}$ is the indicator of damage on a facet of normal h

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The integral is taken over a sphere with radius 1 centered at the considered point and at the fatigue limit the criterion is written as:

$$E_{FB} = 1 \quad (3)$$

2.2 Fatigue limits

Soh Fotsing [8] studied the experimental anisotropy in fatigue of two species of tropical wood Azobe and Bilinga. They predicted that the anisotropy of fatigue limit is based on the Ankinson formula. Applied to the endurance limit in a symmetrical alternating traction, it is defined by:

$$\sigma_{-1}(\gamma) = \frac{\sigma_{-1}(0°)\sigma_{-1}(90°)}{\sigma_{-1}(0°)sin^2(\gamma) + \sigma_{-1}(90°)cos^2(\gamma)} \quad (4)$$

Where, γ is the angle of inclination of the direction considered with respect to the fiber direction

It gives the fatigue limits in accordance with the direction considered viewed in the radial plane of the timber (Fig 1).

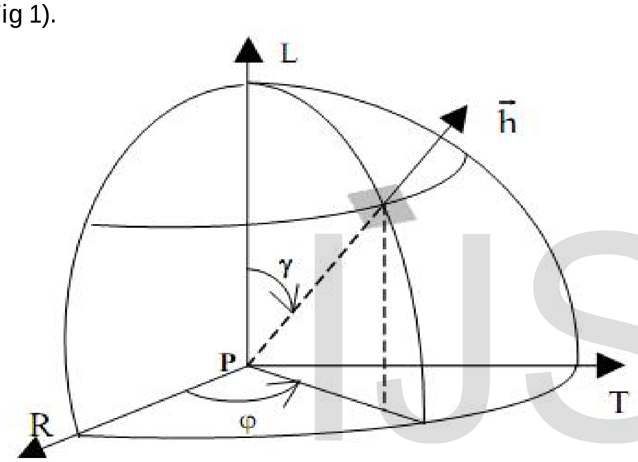

Fig 1 Atracking direction relative to the direction fibers (L) [7]

Further obtaining the fatigue limits at 105 cycles enabled their tracing S-N curves rotating bending in the preferred directions L and R (Fig 2).

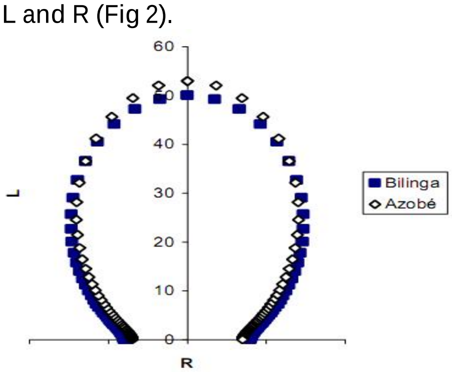

Fig 2 Plane anisotropy curves in the (R, L) fatigue limit to 105 cycles [8]

The particular shape of the curves is a considered characteristic of the anisotropy of both types of wood.

3 Towards an Integration of Anisotropy

3.1. Wood

The orthotropic material and having a rotational symmetry around the fiber direction L (Fig 1) is considered. This, therefore, makes the angle γ solely responsible for the identification of directions in the radial plane (L, R).

The rotational symmetry around L causes the invariance of the fatigue limit $\sigma_{-1}(\varphi, \gamma, N)$ towards φ. Practically, work is done in the plane φ = 0 (that is to say the radial plane (L, R)) and raises $\sigma_{-1}(0, \gamma, N) = \sigma_{-1}(\gamma, N)$, which we simply denote as $\sigma_{-1}(\gamma)$. It will be the same for $\sigma_0(\gamma)$ and $\tau_{-1}(\gamma)$ which will be considered in the same way when it comes to calculating the constants of the FB criterion.

The woods used are Azobé (Lophira alata) and Bilinga (Nauclea diderrichii) whose mechanical characteristics at a humidity rate of 12% [7] are given in table 1 and 2.

Table 1 Static Features Azobé and Bilinga [7]

Species	Azobé	Bilinga
Volumetric masse (g/cm3)	1,04	0,86
Resistance to longitudinal rupture in compression Rm (Kg/cm2)	983	761
Resistance to transverse and tensile rupture Rm (Kg/cm2)	49,5	26,7
Flexural strength (Kg/cm2)	2048	1584
Longitudinal Young modulus E (Kg/cm2)	164000	134000

Table 2 Fatigue characteristics of wood in orthotropic directions L and R [7]

Species	Fatigue limits at 105 cycles (MPa)					
	$\sigma_{-1}(0°)$	$\sigma_{-1}(90°)$	$\sigma_0(0°)$	$\sigma_0(90°)$	$\tau_{-1}(0°)$	$\tau_{-1}(90°)$
Azobe	53	3.5	105	5	24	2.11
Bilinga	50	4	98	6	22	2.6

3.2. Proposal for a new model of anisotropy

The study of fracture mechanics shows that the circular incidence of the geometry of defects characterizes isotropy, while the elliptical characterizes towards a dependence of a direction and thus the anisotropy. The approach consists of establishing a similar principle for the anisotropy of the symmetrical alternating fatigue limit $\sigma_{-1}(\varphi, \gamma, N)$ in the $\vec{h}$ direction.

Our idea consist of postulating that these fatigue limits follow an elliptical form of major and minor axes L and R in the radial plane (L, R). This assumption has a number of advantages:

The elliptical shape is analytic;

It requires only two tests for its implementation: $\sigma_{-1}(0)$ to L and $\sigma_{-1}(90)$ to R;

Its shape fits the formulation of fatigue criteria.

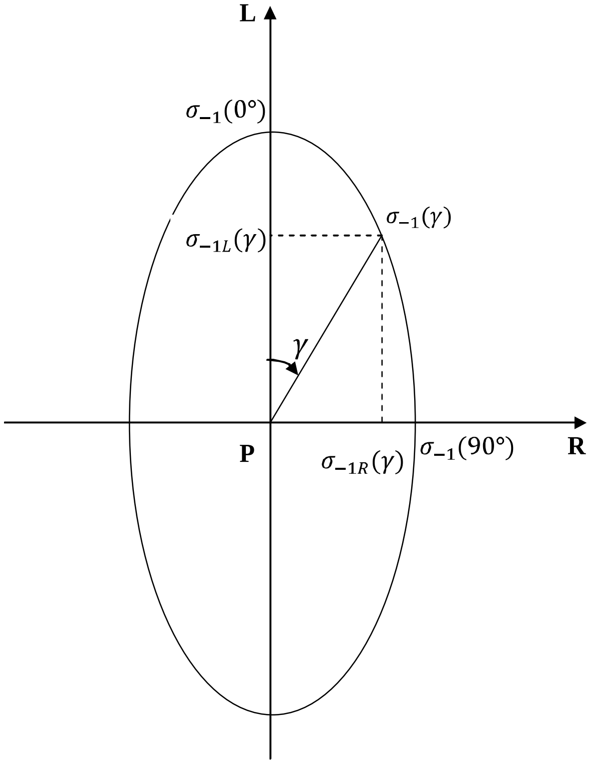

Fig 3 Ellipse anisotropy symmetrical alternated tensile

The values of the major and minor axes are: $2\sigma_{-1}(0)$ and $2\sigma_{-1}(90)$.

For a direction γ with respect to L, the projections of the representative point of the fatigue limit $\sigma_{-1}(\gamma)$ on the axes (L, R) are denoted: $\sigma_{-1L}(\gamma)$ and $\sigma_{-1R}(\gamma)$. We have:

$$\sigma_{-1}(\gamma)\begin{pmatrix}\sigma_{-1L}(\gamma) = \sigma_{-1}(\gamma)\cos\gamma \\ \sigma_{-1R}(\gamma) = \sigma_{-1}(\gamma)\sin\gamma\end{pmatrix}_{(L,R)} \quad (5)$$

In the plan (L, R) the equation of the anisotropy ellipse of stress is written as:

$$\frac{\sigma_{-1L}^2(\gamma)}{\sigma_{-1}^2(0)} + \frac{\sigma_{-1R}^2(\gamma)}{\sigma_{-1}^2(90)} = 1 \quad (6)$$

Finally:

$$\boldsymbol{\sigma_{-1}(\gamma)} = \frac{\boldsymbol{\sigma_{-1}(0)\sigma_{-1}(90)}}{\sqrt{\boldsymbol{\sigma_{-1}^2(90)cos^2(\gamma)+\sigma_{-1}^2(0)sin^2(\gamma)}}} \quad (7)$$

This last relation enables us to calculate the symmetrical alternating fatigue limit of $\sigma_{-1}(\gamma)$ for any given direction $\vec{h}$. We recall here for a comparison purposes the Soh Fotsing [7] relationship inspired by the formula Ankinson (eq 4).

The symmetrical alternating strain limit $\sigma_{-1}(\gamma)$ is the term that appears in the fatigue criterion FB, precisely in the equation of the indicator of damage. This term is replaced by its expression (eq 7) to take into account the anisotropy. The application on wood consist of using fatigue limits, according to equation 7 in two preferred directions longitudinal (L) and radial (R)

4 Anisotropy function

4.1. Isotropic materials

Isotropic materials or assumed isotropic materials have equal fatigue limits in all directions, it would mean that $\sigma_{-1}(0) = \sigma_{-1}(90) = \sigma_{-1}$. Substituting in equation 7, we obtain:

$$\sigma_{-1}(\gamma) = \sigma_{-1} \quad (8)$$

The fatigue limit is no longer a function of γ. The set of points in the plane (L, R) satisfying this equality is the circle with center O and radius σ_{-1}shown in Fig 4.

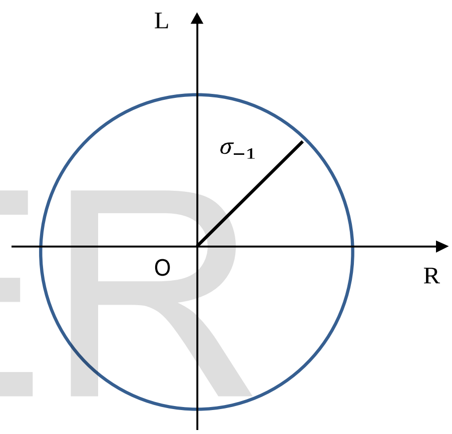

Fig 4 Curve isotropy

We have a similar curve (of the same radius σ_{-1}) with the anisotropy function of Soh Fotsing [7] inspired by the formula Ankinson function.

4.2. Anisotropic materials

4.2.1 Comparison function Soh Fotsing

The equation 4 of Soh Fotsing [7] gives us the orthotropic plane (L, R) in Fig 4. The curve obtained from the elliptical anisotropy function (eq 7) is shown in Fig 5.

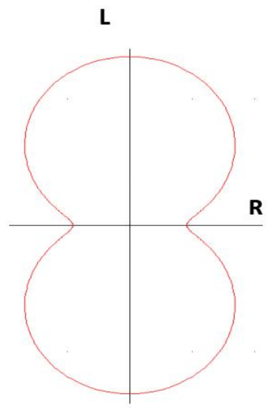

Fig 4 Anisotropy curve for Azobé, formula Fotsing Soh [7]

Two points coincide with the Soh Fotsing [7] curve: the fatigue limit in the L direction and the fatigue limit in the R direction. This is due to the use of strain limits determined by the former. Therefore in order to generate our anisotropy, curve we must start from two points to satisfy equation 7. Equations (4) and (7) have identical values for $\gamma = 0°$ and $\gamma = 90°$ (i.e. in the directions L and R).

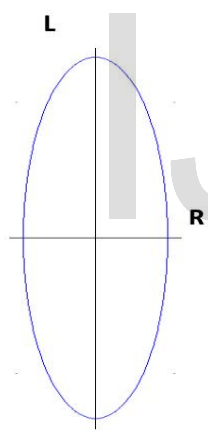

Fig 5 Anisotropic curve of Azobe

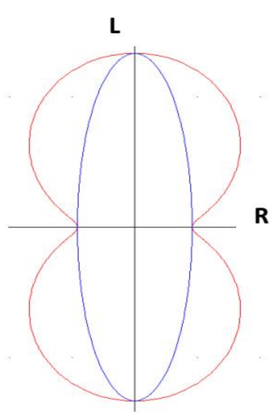

Fig 6 Anisotropic curves of Azobe (fig 4 and fig 5)

The values of the fatigue limits at 45 ° of the Soh Fotsing function and ours are respectively 6.57 MPa and 4.94 MPa.

4.2.2. Comparison of fatigue limits of Azobe and Bilinga

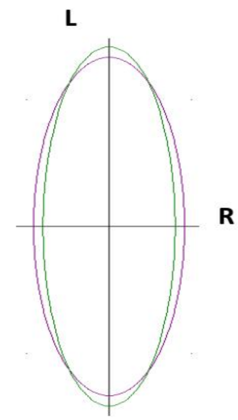

Fig 7 Curves of anisotropy of Azobe (purple) and Bilinga (green)

We observe the Azobe and Bilinga curves in Fig 7. This shows that the two curves have a similar behavior. We also observe that the Azobe (red) has fatigue limits lower than those of Bilinga in areas close to 90 °. In the blue area (represented by a blue ellipse) their behaviors get similar and overlap at an angle α and a value of $\sigma_{-1}(\alpha)$ defined below.

Setting the fatigue limits of Azobe and Bilinga at an angle α, we have:

$$\frac{53*3,5}{\sqrt{3,5^2 cos^2(\alpha)+53^2 sin^2(\alpha)}} = \frac{50*4}{\sqrt{4^2 cos^2(\alpha)+50^2 sin^2(\alpha)}} \quad (9)$$

We obtain: $\tan\alpha = 0{,}048$

Where $\boldsymbol{\alpha} \cong \mathbf{2,75°}$ and $\boldsymbol{\sigma_{-1}(2,75°)} \cong \mathbf{42,92}\ \boldsymbol{MPa}$ (10)

In any plane (L, T) the anisotropy function is identical and therefore does not depend on the angle φ. The assumption made above that the characteristics of orthotropic wood does not depend on the angle φ is verified.

5 RESULTATS AND DISCUSSION

By integrating the elliptic function of anisotropy, equation 7, the fatigue criterion Fogue-Bahaud, equation 1, we obtain the following system:

$$\begin{cases} xa^2 + yb^2 + zab = e \\ ua^2 + vb^2 + wab = 1 \\ xa^2 + y(b+d)^2 + za(b+d) = g \end{cases} \quad (11)$$

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with a, b and d, the coefficients of the damage indicator E_h of FB criterion and x, y, z, u, v, w, e and g of the constants defined below:

$$x = \frac{6}{105\sigma_{-1}^2(0)} + \frac{8}{105\sigma_{-1}^2(90)} \; ; \; y = \frac{15}{105\sigma_{-1}^2(0)} + \frac{6}{105\sigma_{-1}^2(90)} \; ; \; z = \frac{16}{105\sigma_{-1}^2(0)} + \frac{12}{105\sigma_{-1}^2(90)}$$

$$u = \tau_{-1}^2(0)\left(\frac{10}{105\sigma_{-1}^2(0)} + \frac{30}{105\sigma_{-1}^2(90)}\right) ; \; v = \tau_{-1}^2(0)\left(\frac{4}{105\sigma_{-1}^2(0)} + \frac{24}{105\sigma_{-1}^2(90)}\right) ; \; u = \tau_{-1}^2(0)\left(\frac{13}{140\sigma_{-1}^2(0)} + \frac{157}{420\sigma_{-1}^2(90)}\right) ; \; e = \frac{1}{\sigma_{-1}^2(0)} \; ; \; g = \frac{4}{\sigma_0^2(0)}$$

The equations of the system not bounded depends on the first and then the second equation.

$$\begin{cases} xa^2 + yb^2 + zab = e \\ ua^2 + vb^2 + wab = 1 \end{cases} \qquad (12)$$

Solving this system gives us four solutions that gives all imaginary values. However, weight of the indicator of damage are constants that must not be imaginary or negative. An explanation for this result is that the model is not suitable for wood material. Recall that Soh Fotsing [7] is located in the worst case possible for the application of the test FB. This would mean that only the fatigue limits in the direction of 90 ° was used to calculate the limits of validity of the test. But the fatigue limits in the direction 0 ° were also used in the calculations of the constants of the test.

One of the important remarks is that the test is pessimistic as long as the predicted values are inside of the curve modeling the experimental points (figure 6). Which gives it a security nature. In the security domain, the material will remain under stress without anyone caring about its fatigue.

6 CONCLUSION

At the end of our investigation, characterization of wood material confirms a highly anisotropic behavior. This behavior is modeled through the fatigue limits of the material. These belong to a closed curve called anisotropy ellipse.

The study of the anisotropy ellipse shows that the isotropy is a limit for general cases of anisotropy. The equations of the ellipse and the anisotropy Soh Fotsing produced results more or less accurate depending on the material. The behavior of two species of wood studied superposed at an angle of 2.75 °. The elliptical equation predicts the anisotropy with two parameters. This is very good, since the idea is to predict the anisotropy with less parameters.

Redefining the limits of validity of Fogue-Bahaud criterion or application to other materials also allow a better assessment of the capacity of the elliptic function to predict the anisotropy. To refine this work, it would be interesting to reduce the security domain. That is to say the change of anisotropy ellipse so that it approximates the curve Soh Fotsing (experimental points).

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